Period 1, Oct 16, 2024

15. $y = \sin xy$ $\Rightarrow y = \sin u$

$u = xy$ $\quad \frac{dy}{du} = \cos u$

$\frac{du}{dx} = 1 \cdot y + x \cdot \frac{dy}{dx}$

$\frac{du}{dx} \cdot \frac{dy}{du} = (y + x\frac{dy}{dx})(\cos u)$

$\frac{dy}{dx} = (y + x\frac{dy}{dx})(\cos xy)$

$\frac{dy}{dx} = y\cos xy + x\frac{dy}{dx}\cos xy$

$-x\frac{dy}{dx}\cos xy$ $\quad -x\frac{dy}{dx}\cos xy$

$\frac{dy}{dx} - x\frac{dy}{dx}\cos xy = y\cos xy$

$\frac{dy}{dx}(1 - x\cos xy) = y\cos xy$

$\frac{dy}{dx} = \frac{y\cos xy}{1 - x\cos xy}$

7. $x^3y^3 - y = x$

$3x^2y^3 + x^3 \cdot 3y^2\frac{dy}{dx} - \frac{dy}{dx} = 1$

$-3x^2y^3$ $\quad -3x^2y^3$

$3x^3y^2\frac{dx}{dx} - \frac{dy}{dx} = 1 - 3x^2y^3$

$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$

$3x^3y^2 - 1$ $\quad 3x^3y^2 - 1$

$\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$

7. $(1 - 3x^2y^3)/(3x^3y^2 - 1)$

In Exercises 17–20, (a) find two explicit functions by solving the equation for y in terms of x, (b) sketch the graph of the equation and label the parts given by the corresponding explicit functions, (c) differentiate the explicit functions, and (d) find dy/dx implicitly and show that the result is equivalent to that of part (c).

17. $x^2 + y^2 = 64$

18. $x^2 + y^2 - 4x + 6y + 9 = 0$

19. $16x^2 + 25y^2 = 400$

20. $16y^2 - x^2 = 16$

$x^2 + y^2 = 64$

$2x + 2y\frac{dx}{dx} = 0$

$2y\frac{dy}{dx} = -2x$ $\quad 2y$ $\quad 2y$

$\frac{dy}{dx} = \frac{-x}{y} = \frac{-x}{\pm\sqrt{64 - x^2}}$

17)

$x^2 + y^2 = 64$

$-x^2$ $\quad -x^2$

$\sqrt{y^2} = \sqrt{64 - x^2}$

$y = \pm\sqrt{64 - x^2} = (64 - x^2)^{\frac{1}{2}}$

$u = 64 - x^2$ $\quad y = \pm u^{\frac{1}{2}}$

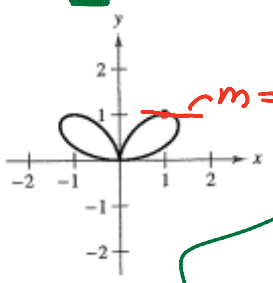
$\frac{du}{dx} = -2x$ $\quad \frac{dy}{du} = \pm\frac{1}{2\sqrt{u}}$

$\frac{du}{dx} \cdot \frac{dy}{du} = (-2x) \cdot \left(\pm\frac{1}{2\sqrt{64 - x^2}}\right)$

31. Bifolium:

$(x^2 + y^2)^2 = 4x^2y$

Point: (1, 1)



$m = 0$

$(x^2+y^2)^2 = 4x^2y$

$2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) =$

$y = (x^2+y^2)^2$

$u = x^2+y^2$

$\frac{du}{dx} = 2x + 2y\frac{dy}{dx}$

$y = u^2$

$\frac{dy}{du} = 2u$

$\frac{dy}{du}\cdot\frac{du}{dx} = \frac{dy}{dx} = 2u(2x+2y\frac{dy}{dx})$

$2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 8x\cdot y + 4x^2\cdot\frac{dy}{dx}$

$2(1^2+1^2)\left(2\cdot 1+2\cdot 1\cdot\frac{dy}{dx}\right) = 8\cdot 1\cdot 1 + 4\cdot 1^2\frac{dy}{dx}$

$4\left(2+2\frac{dy}{dx}\right) = 8 + 4\frac{dy}{dx}$

$8 + 8\frac{dy}{dx} = 8 + 4\frac{dy}{dx} \Rightarrow 8\frac{dy}{dx} = 4\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 0$

In Exercises 45–50, find d^2y/dx^2 in terms of x and y.

45. $x^2 + y^2 = 4$

46. $x^2y^2 - 2x = 3$

47. $x^2 - y^2 = 36$

48. $1 - xy = x - y$

49. $y^2 = x^3$

50. $y^2 = 10x$

$\frac{-y}{y^2} + \frac{-x^2}{y^2} = \frac{-1}{y} - \frac{x^2}{y^3}$

45) $x^2 + y^2 = 4$

$2x + 2y\frac{dy}{dx} = 0$

$-2x \quad -2x$

$2y\frac{dy}{dx} = -2x$

$\frac{2y}{2y}$ $\quad \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-x}{y}$

$\frac{d^2y}{dx^2} = \frac{-1\cdot y - (-x)\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y + x\cdot\left(\frac{-x}{y}\right)}{y^2} = \frac{\frac{-y^2}{y} - \frac{x^2}{y}}{y^2} = \frac{\frac{-y^2-x^2}{y}}{\frac{y^2}{1}}$

$\frac{-y^2-x^2}{y}\cdot\frac{1}{y^2} = \frac{-y^2-x^2}{y^3} = \frac{-1(y^2+x^2)}{y^3} = \frac{-1\cdot 4}{y^3}$

47) $x^2 - y^2 = 36$

$2x - 2y\frac{dy}{dx} = 0$

$+2y\frac{dy}{dx} \quad +2y\frac{dy}{dx}$

$\frac{2x}{2y} = \frac{2y\frac{dy}{dx}}{2y}$

$\frac{x}{y} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{x}{y}$

$\frac{d^2y}{dx^2} = \frac{1\cdot y - x\cdot\frac{dy}{dx}}{(y)^2}$

$\frac{d^2y}{dx^2} = \frac{\frac{y}{1}\cdot\frac{y}{y} - x\cdot\frac{x}{y}}{y^2} = \frac{\frac{y^2}{y} - \frac{x^2}{y}}{\frac{y^2}{1}} = \frac{\frac{y^2-x^2}{y}}{\frac{y^2}{1}}$

$\frac{d^2y}{dx^2} = \frac{y^2-x^2}{y}\cdot\frac{1}{y^2} = \frac{y^2-x^2}{y^3} = \frac{-1(x^2-y^2)}{y^3}$

$\frac{d^2y}{dx^2} = \frac{-1(36)}{y^3}$

$y^2 = x^3$

$\frac{2y\frac{dy}{dx}}{2y} = \frac{3x^2}{2y}$

$\frac{dy}{dx} = \frac{3x^2}{2y}$

$\frac{d^2y}{dx^2} = \frac{6x\cdot 2y - 3x^2\cdot 2\frac{dy}{dx}}{(2y)^2}$

$\frac{d^2y}{dx^2} = \frac{12xy - 6x^2\left(\frac{3x^2}{2y}\right)}{4y^2}$

$\frac{d^2y}{dx^2} = \frac{\frac{12xy\cdot 2y}{1\cdot 2y} - \frac{18x^4}{2y}}{4y^2}$

$\frac{d^2y}{dx^2} = \frac{\frac{24xy^2}{2y} - \frac{18x^4}{2y}}{4y^2} = \frac{\frac{24xy^2-18x^4}{2y}}{\frac{4y^2}{1}}$

$\frac{d^2y}{dx^2} = \frac{24xy^2-18x^4}{2y}\cdot\frac{1}{4y^2}$

$\frac{d^2y}{dx^2} = \frac{24xy^2-18x\cdot x^3}{8y^3}$

$\frac{d^2y}{dx^2} = \frac{24xy^2-18x\cdot y^2}{8y^3}$

$\frac{d^2y}{dx^2} = \frac{6xy^2}{8y^3} = \frac{3x}{4y}$

25. $x^{2/3} + y^{2/3} = 5, \quad (8, 1)$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$

$\frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}}\frac{dy}{dx} = 0$

$\frac{2}{3\sqrt[3]{8}} + \frac{2}{3\sqrt[3]{1}}\frac{dy}{dx} = 0$

$\frac{2}{3\cdot 2} + \frac{2}{3\cdot 1}\frac{dy}{dx} = 0$ $\quad -\frac{1}{3}$

$-\frac{1}{3}$

$\frac{3}{2}\cdot\frac{2}{3}\frac{dy}{dx} = -\frac{1}{3}\cdot\frac{3}{2} = -\frac{1}{2}$

27. $\tan(x + y) = x, \quad (0, 0)$

$\tan(x+y) = y$

$u = x+y$

$\frac{du}{dx} = 1 + \frac{dy}{dx}$

$\tan u = y$

$\sec^2 u = \frac{dy}{du}$

$\frac{dx}{dx}\cdot\frac{dy}{du} = \frac{dy}{dx}$

$\left(1+\frac{dy}{dx}\right)(\sec^2(x+y)) = \frac{dy}{dx}$

$\tan(x+y) = x$

$\left(1+\frac{dy}{dx}\right)\sec^2(x+y) = 1$

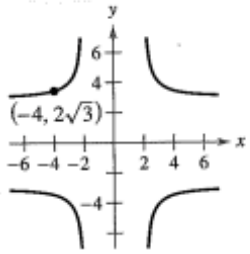
$\left(1+\frac{dy}{dx}\right)\cdot\sec^2(0+0) = 1$

$\sec 0 = 1$

$\left(1+\frac{dy}{dx}\right)^2 = 1$

$1+\frac{dy}{dx} = 1$

37. Cruciform



$x^2y^2 - 9x^2 - 4y^2 = 0$

Product Rule

$2xy^2 + x^2\cdot 2y\frac{dy}{dx} - 18x - 8y\frac{dy}{dx} = 0$

$-2x^2y\frac{dy}{dx}$ $\quad +8y\frac{dy}{dx}$ $\quad +8y\frac{dy}{dx}$ $\quad -2x^2y\frac{dy}{dx}$

$2xy^2 - 18x = 8y\frac{dy}{dx} - 2x^2y\frac{dy}{dx}$

$\frac{2xy^2 - 18x}{(8y - 2x^2y)} = \frac{dy}{dx}\frac{(8y - 2x^2y)}{(8y - 2x^2y)}$

$\frac{2xy^2 - 18x}{8y - 2x^2y} = \frac{dy}{dx}$

$m = \frac{1}{2\sqrt{3}}$

Point $(-4, 2\sqrt{3})$

$y - 2\sqrt{3} = \frac{1}{2\sqrt{3}}(x+4)$

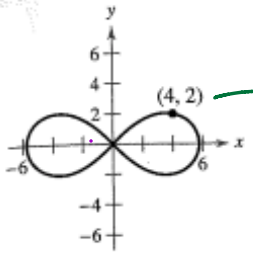
$y - 2\sqrt{3} = \frac{\sqrt{3}}{6}(x+4)$

$\frac{2(-4)(2\sqrt{3})^2 - 18(-4)}{8(2\sqrt{3}) - 2(-4)^2\cdot 2\sqrt{3}} = \frac{dy}{dx} = \frac{-96+72}{-24\cdot 2\sqrt{3}} = \frac{-24}{-24\cdot 2\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

12

8 $\quad$ 32

39. Lemniscate $3(x^2+y^2)^2 = 100(x^2-y^2)$



$3(x^2+y^2)^2 = 100x^2 - 100y^2$

$3 \cdot 2(x^2+y^2)\left(2x + 2y\frac{dy}{dx}\right) = 200x - 200y\frac{dy}{dx}$

$6(4^2+2^2)\left(2\cdot 4 + 2\cdot 2\frac{dy}{dx}\right) = 200\cdot 4 - 200\cdot 2\frac{dy}{dx}$

$20 \quad 8 \quad 4 \quad 800 \quad 400$

$120\left(8 + 4\frac{dy}{dx}\right) = 800 - 400\frac{dy}{dx}$

$960 + 480\frac{dy}{dx} = 800 - 400\frac{dy}{dx}$

$-960 \quad +400\frac{dy}{dx} \quad -960 \quad +400\frac{dy}{dx}$

$880\frac{dy}{dx} = -160$

$\frac{880}{880}$

$\frac{dy}{dx} = \frac{-2\cdot 80}{11\cdot 80}$

$m = -\frac{2}{11} \quad (4,2)$

$y - 2 = -\frac{2}{11}(x-4)$

$y - 2 = -\frac{2}{11}x + \frac{8}{11} + \frac{22}{11}$

$+2$

$y = -\frac{2}{11}x + \frac{30}{11}$

$y = (x^2+y^2)^2$

$u = x^2+y^2$

$\frac{du}{dx} = 2x + 2y\frac{dy}{dx}$

$y = u^2$

$\frac{dy}{du} = 2u$

$\frac{du}{dx} \cdot \frac{dy}{du}$

$\left(2x + 2y\frac{dy}{dx}\right)(2u)$

$\left(2x + 2y\frac{dy}{dx}\right)\cdot 2(x^2+y^2)$